Attosecond electromagnetic pulse generation due to the interaction of a relativistic soliton with a breaking-wake plasma wave

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During the interaction of a low-frequency relativistic soliton with the electron density modulations of a wake plasma wave, part of the electromagnetic energy of the soliton is reflected in the form of an extremely short and ultraintense electromagnetic pulse. We calculate the spectra of the reflected and of the transmitted electromagnetic pulses analytically. The reflected wave has the form of a single cycle attosecond pulse.

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I. INTRODUCTION

Over the last few years we have witnessed a very significant progress in the generation of ultraintense laser pulses. Present day lasers produce pulses with intensities that approach $10^{22}\ \mathrm{W/cm^2}$ [1]. In the so-called relativistic regime, where the quiver energy of the electrons is equal to, or greater than, their rest-mass energy, i.e., where the laser light intensity exceeds $I \sim 10^{18}\ \mathrm{W/cm^2}$, interesting nonlinear properties of the laser-plasma interaction come into play (see e.g., the review articles [1,2], and the literature quoted therein). Among the rich variety of nonlinear processes that accompany the electromagnetic (e.m.) pulse propagation through a tenuous plasma, in the present paper we shall address the Langmuir waves [3,4] and the relativistic e.m. solitons [5–8] that are left in the wake behind the laser pulse.

As is well known, the use of the collective electric fields in plasmas in the laser wake-field accelerator (LWFA) provides one of the most promising approaches to high-performance compact electron accelerators [4]. The wake-field acceleration of electrons has been observed in the experiments reported in Ref. [9] (see also the review article [10]). Another use of the wake field is in the intensification of co-propagating short laser pulses, known as the photon accelerator [11].

In Ref. [12] a method for generating ultrahigh-intensity e.m. fields was proposed, based on the compression of a laser pulse, the up-shifting of its carrier frequency, and the pulse focusing by a counterpropagating, breaking plasma wave. In this case the electron density modulations in the wake wave act as parabolic relativistic flying mirrors. This method makes it possible to achieve the critical field of quantum electrodynamics (known as the Schwinger limit [13]) with present-day laser systems. Below, we shall show that the role of the laser pulse can be taken by a relativistic e.m. soliton. As is well known, for a long time solitons have attracted great attention because of their resilient, robust behavior [14] and the research field on solitons has grown enormously. This research topic ranges from the nonlinear wave dynamics in shallow water, where solitons have been discovered (see, e.g., Refs. [14,15]), to quantum field theory [16]. Relativistic e.m. solitons provide an example of coherent structures and represent a fundamental feature of the nonlinear laser-plasma interaction. As was shown in Refs. [5–8], a significant fraction of the order of 30–40 % of the laser pulse energy can be trapped in these structures in the form of e.m. energy oscillating at a frequency below the Langmuir frequency $\omega_{pe} = (4\pi n e^2/m_e)^{1/2}$ of the surrounding plasma. The typical size of these solitons is of the order of the collisionless electron skin depth $d_e = c/\omega_{pe}$. The e.m. fields inside the solitons consist of synchronously oscillating electric and magnetic fields plus a steady electrostatic field which arises from charge separation as electrons are pushed outward by the ponderomotive force of the oscillating fields. On a long time scale, when the effects of the ion motion become important, the ponderomotive force forms cavities in the plasma density, which have been named post solitons [17]. Post solitons were observed experimentally in Ref. [18] with the use of the proton imaging technique [19]. However, for simplicity, in the present paper we shall consider conditions when the effects of the ion motion can be neglected. We shall regard the soliton in the reference frame co-moving with the wake wave as a semicycle e.m. wave packet. As a result of the packet interaction with the electron density modulations in the wake wave, a portion of its energy is reflected in the direction of propagation of the wake plasma wave. This results in the transformation of the low frequency soliton field into a high frequency ultrashort e.m. burst. In a tenuous plasma the frequency up-shift can be so large that it can provide a different mechanism for generating attosecond pulses, distinct from the mechanisms described in the literature [20]. In this paper we limit our scope to the analytical investigation of this mechanism in a simplified one-dimensional geometry. A numerical investigation based on particle-in-cell (PIC) simulations in a more realistic geometry will be the subject of a forthcoming paper.

The present paper is organized as follows. In the next section we rederive the relationships between the frequency, wave number, and amplitude of the incident pulse propagating in an underdense plasma and of the pulse reflected at the relativistic mirror. In Sec. III we analyze the interaction of the e.m. soliton with the electron density slab formed by a breaking Langmuir wave. In the final section we discuss the results obtained and present the conclusions.

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II. REFLECTION OF THE ELECTROMAGNETIC WAVE AT THE RELATIVISTIC MIRROR

Let us consider the reflection of an e.m. wave at a relativistic mirror in the case when the wave propagates through an underdense plasma. In this case the wave frequency ω_0 and wave number k_0 are related by the dispersion equation

$$\omega_0^2 = k_0^2 c^2 + \omega_{pe}^2. \tag{1}$$

The Langmuir frequency ω_{pe} is a relativistic invariant, i.e., it does not change under Lorentz transformations.

The linearly polarized incident wave propagates from the right to the left, i.e., its wave four-vector is $K=(\omega_0/c, -k_0, 0, 0)$, and the mirror moves from the left to the right with the phase velocity $V=(v_{ph}, 0, 0)$ of the breaking plasma wave. As is well known [4], the phase velocity $v_{ph}=\beta_{ph}c$ of the wake field is equal to the laser pulse group velocity, which is close to the speed of light in vacuum if the laser pulse propagates in an underdense plasma. The vector potential of the incident wave is given by

$$a_{\mathrm{in}}(x,t) = a_0 \sin(\omega_0 t + k_0 x), \tag{2}$$

where $k_0=(\omega_0^2-\omega_{pe}^2)^{1/2}/c$ and $a_0=eE_0/m_e\omega_0 c$ with E_0 the amplitude of the the electric field, m_e and e the electron mass and charge, and c the speed of light in vacuum. Performing a Lorentz boost to the reference frame where the mirror is at rest (denoted as the **M** frame), we obtain for the vector potential of the wave,

$$a_{\mathrm{in}}(x',t') = a_0 \sin(\omega' t' + k' x'), \tag{3}$$

where

$$\omega' = \gamma_{ph}(\omega_0 + k_0 v_{ph}), \quad k' = -\gamma_{ph}(k_0 + \omega_0 \beta_{ph})/c, \tag{4}$$

$$t' = \gamma_{ph}(t - x\beta_{ph}/c), \quad x' = \gamma_{ph}(x - t v_{ph}), \tag{5}$$

$\beta_{ph}=v_{ph}/c$ and $\gamma_{ph}=(1-\beta_{ph}^2)^{1/2}$. We notice that the transverse component of the vector potential is a relativistic invariant so that a_0 does not change under the Lorentz transformation. Assuming ideal reflection, we write for the reflected e.m. wave in the reference frame **M**

$$a_{\mathrm{ref}}(x',t') = -a_0 \sin(\omega' t' - k' x'). \tag{6}$$

Now we transform back to the laboratory frame **L** and obtain for the reflected e.m. wave

$$a_{\mathrm{ref}}(x,t) = -a_0 \sin(\omega'' t - k'' x), \tag{7}$$

where the frequency ω'' of the reflected wave is equal to

$$\omega'' = \gamma_{ph}(\omega' - k' v_{ph}) = \gamma_{ph}^2\left[\omega_0(1+\beta_{ph}^2) + 2k_0 v_{ph}\right]. \tag{8}$$

In the case of the e.m. wave propagating in vacuum, using the dispersion relation $\omega_0^2=k_0^2c^2$, we obtain from Eq. (8), the Einstein relationship $\omega''=\omega_0(1+\beta_{ph})/(1-\beta_{ph})\approx 4\omega_0\gamma_{ph}^2$ for $\beta_{ph}\to 1$ (see Refs. [23,24]). On the contrary, for the e.m. wave in a plasma, we must use the dispersion relation (1) which yields

$$\omega'' = \gamma_{ph}^2\left[\omega_0(1+\beta_{ph}^2) + 2(\omega_0^2 - \omega_{pe}^2)^{1/2}\beta_{ph}\right]. \tag{9}$$

In the limit $\omega_0^2 \gg \omega_{pe}^2$ we obtain the vacuum frequency upshift: $\omega''=\omega_0(1+\beta_{ph})/(1-\beta_{ph})$, while in the case $\omega_0^2=\omega_{pe}^2$ we find

$$\omega'' = \omega_0\frac{(1+\beta_{ph}^2)}{(1-\beta_{ph}^2)} \equiv \omega_{pe}\frac{(1+\beta_{ph}^2)}{(1-\beta_{ph}^2)} \approx 2\omega_{pe}\gamma_{ph}^2. \tag{10}$$

This limit corresponds to the case of the soliton reflection at the mirror.

III. SOLITON INTERACTION WITH A BREAKING-WAKE PLASMA WAVE

When an intense pulse interacts with a plasma, it forces the plasma electrons to move with relativistic velocities. In turn, this motion induces a wake field in the plasma. The nonlinearity of the strong wake field leads to a nonlinear wave profile and in particular to the steepening of the wave and to the formation of sharply localized maxima, "spikes", in the electron density [21]. This means that the wake field enters the wave-breaking regime (see Ref. [2] references therein). Theoretically, the electron density in the spikes tends to infinity, but remains integrable [2]. From the continuity equation it follows that the electron density is given as a function of $X=x-v_{ph}t$ by

$$n_e(X) = \frac{n_0\beta_{ph}}{\beta_{ph} - \beta_u(X)}, \tag{11}$$

where n_0 is the ion concentration in the plasma and the speed of the electrons (divided by the speed of light) β_u varies from $-\beta_{ph}$ to β_{ph}. As a consequence, the electron density tends to infinity at the breaking points and it is of the order of $n_0/2$ in the regions in between. Thus, close to the wave breaking conditions, we can use the approximate form of the electron density,

$$n_e(X) = \frac{n_0}{2}[1 + \lambda_p \delta(X)], \tag{12}$$

instead of Eq. (11). Here λ_p is the wavelength in the wave breaking regime. The density spike in Eq. (12) can be expected to partially reflect a counterpropagating e.m. wave.

In this section we study the interaction of a one dimensional soliton with the wake field (12). We recall that relativistic plasma solitons are formed during the interaction of a high intensity laser pulse with the plasma. This interaction causes the laser pulse to lose its energy which is transformed into the energy of various plasma modes and into kinetic energy of the charged particles. As the pulse propagates in the plasma, the number of photons in the pulse is approximately conserved [8]. This fact, along with the loss of pulse energy, leads to the decrease of frequency of the pulse down to $\omega_{\mathrm{cr}}=\omega_{pe}$. As a result, the part of the pulse energy becomes trapped in electron density cavities in the form of low-frequency radiation.

Let us assume that the soliton is formed by a circularly polarized laser pulse. In order to describe the transverse components of vector potential, we introduce the complex dimensionless function A,

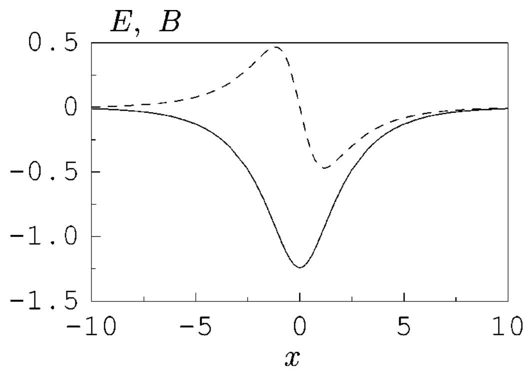

FIG. 1. The dependence of the electric (solid line) and magnetic (dashed line) fields of the initial soliton on x in the laboratory frame; x is measured in units c/ω_{pe}; time is set to zero, $t=0$, and $\omega=0.85\omega_{pe}$.

$$A=\frac{e}{m_e c^2}(A_y+iA_z). \tag{13}$$

Using the requirement that the vector potential vanishes at infinity, one can write the stationary solution for the system of the cold hydrodynamic electron equations and of Maxwell's equations [6] in the factorized form

$$A(x,t)=\frac{2\varepsilon(\omega_0)\cosh[\varepsilon(\omega_0)\omega_{pe}x/c]e^{i\omega_0 t}}{\cosh^2[\varepsilon(\omega_0)\omega_{pe}x/c]-\varepsilon^2(\omega_0)}\equiv\mathcal{C}(x)e^{i\omega_0 t}, \tag{14}$$

where $\omega_0<\omega_{pe}$ is the soliton frequency, and $\varepsilon(\omega_0)=(1-\omega_{pe}^2/\omega_0^2)^{1/2}$. The form of the electric and magnetic fields inside the soliton is shown in Fig. 1. In the limit of small soliton amplitude, when $\omega_0\approx\omega_{pe}$, i.e., $\varepsilon(\omega_0)\ll 1$, the amplitude $\mathcal{C}(x)$ reduces to $\mathcal{C}(x)\approx 2\varepsilon(\omega_0)/\cosh[\varepsilon(\omega_0)\omega_{pe}x/c]$. This limit provides an example of a soliton described by the so called nonlinear Schrödinger equation (see Refs. [14,15,22]).

Below we consider the interaction of the soliton with the electron density modulation moving with relativistic velocity in the wake plasma wave. The velocity of the wake field is close to the velocity of light in vacuum and is directed along the x axis. In order to simplify the calculations we perform the Lorentz transformation to the reference frame **M**, where the wake plasma wave is at rest and the soliton appears as an e.m. wave packet incident from the right to the left. In this reference frame the vector potential of the soliton, denoted $A'_{\rm in}(x',t')$ where $x=\gamma_{ph}(x'+v_{ph}t')$ and $t=\gamma_{ph}(t'+v_{ph}x'/c^2)$ is given by

$$A'_{\rm in}(x',t')=\mathcal{C}(\gamma_{ph}(x'+v_{ph}t'))e^{i\omega_0\gamma_{ph}(t'+v_{ph}x'/c^2)}. \tag{15}$$

At the mirror, which is assumed to be localized at $x'=0$, the incident e.m. field depends on time as

$$A'_{\rm in}(0,t')=\mathcal{C}(\gamma_{ph}v_{ph}t')e^{i\omega_0\gamma_{ph}t'}. \tag{16}$$

By performing the Fourier transform of the incident pulse, we obtain

$$A'_{\rm in}(0,\omega')=\frac{1}{2\pi}\int_{-\infty}^{\infty}A'_{\rm in}(0,\tau)e^{i\omega'\tau}d\tau. \tag{17}$$

In order to calculate the reflection and the transmission coefficients we should consider the interaction of the e.m. wave with the maximum of the electron density in the breaking Langmuir wave. Since in the wave breaking regime the electron density can be described by the expression (12), this problem is equivalent to the scattering from a delta-function potential. In this approach it is assumed that the continuum model for the electron response remains applicable even accounting for the up-shift of the frequency of the reflected radiation.

Then the amplitudes of the reflected and of the transmitted plane waves are given by the reflection and transmission coefficients [12]:

$$\rho(\omega')=-\frac{q}{q-i\omega'},\quad \tau(\omega')=\frac{i\omega'}{q-i\omega'}, \tag{18}$$

where $q=2\omega_{pe}(2\gamma_{ph})^{1/2}$. The reflected e.m. pulse is given by

$$A'_{\rm ref}(x',t')=\frac{1}{2\pi}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\rho(\omega')A'_{\rm in}(0,\tau)e^{i\omega'\tau}e^{-i\omega'(t'-x'/c)}d\omega' d\tau, \tag{19}$$

where we assumed that the reflected pulse moves with the speed of light and that it is not affected by plasma. This is justified by the fact that the frequency of the reflected pulse is well above the Langmuir frequency so that we can neglect the small difference between the group (and phase) velocity of the pulse and the speed of light in vacuum. Using the relationship

$$\int_{-\infty}^{\infty}\rho(\omega')e^{-i\alpha\omega'}d\omega'=-2\pi q\theta(\alpha)e^{-q\alpha}, \tag{20}$$

where $\theta(\alpha)$ is the theta function $\theta(\alpha)=1$, if $\alpha\geqslant 0$ and $\theta(\alpha)=0$, if $\alpha<0$, we carry out the integration in Eq. (19) over ω':

$$A'_{\rm ref}(x',t')=-qe^{-q(t'-x'/c)}\int_{-\infty}^{t'-x'/c}\mathcal{C}(\gamma_{ph}v_{ph}\tau)e^{(q+i\omega_0\gamma_{ph})\tau}d\tau, \tag{21}$$

where we have absorbed the theta function in the redefinition of the upper limit of integration.

In order to find the expression of the vector potentials of the reflected pulse in the laboratory reference frame **L** we perform a Lorentz transformation and obtain

$$A_{\rm ref}(x,t)=-qe^{-q(1+\beta_{ph})\gamma_{ph}(t-x/c)}\times\int_{-\infty}^{(1+\beta_{ph})\gamma_{ph}(t-x/c)}\mathcal{C}(\gamma_{ph}v_{ph}\tau)e^{(q+i\omega_0\gamma_{ph})\tau}d\tau. \tag{22}$$

Now we introduce two complex functions:

$$E=E_y+iE_z,\quad B=B_y+iB_z, \tag{23}$$

where E_y,E_z,B_y,B_z are the transverse components of the electric and magnetic fields in the reflected e.m. wave. The electric and the magnetic fields are expressed in terms of the vector-potential as

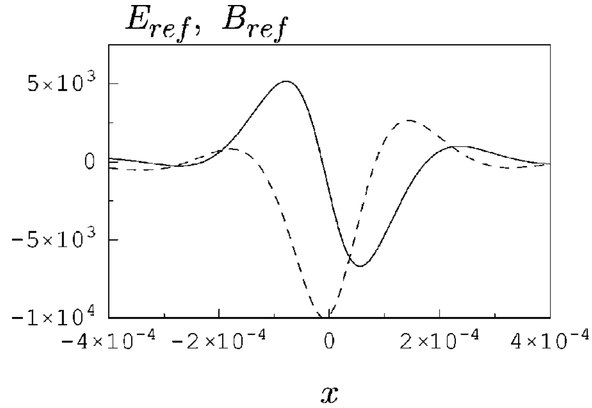

FIG. 2. The dependence of the electric (solid line) and magnetic (dashed line) fields of reflected pulse on x in the laboratory frame; x is measured in units c/ω_{pe}; time is set to zero, $t=0$, $\omega=0.85\omega_{pe}$, and $\gamma_{ph}=100$.

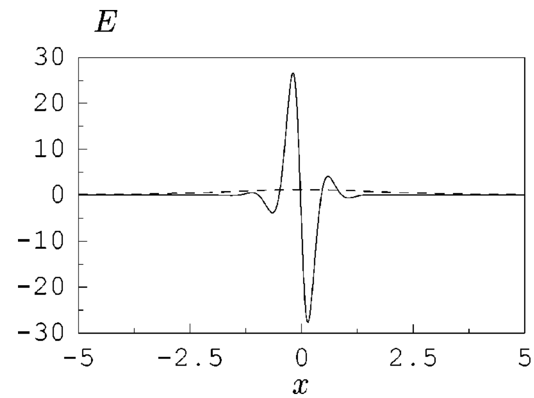

FIG. 3. The dependence of the electric field in the incident (dashed line) and reflected (solid line) pulse on x in the laboratory frame; x is measured in units c/ω_{pe}; time is set to zero, $t=0$, $\omega=0.85\omega_{pe}$, and $\gamma=2$.

$$E=-\frac{1}{c}\frac{\partial A}{\partial t}, \quad B=i\frac{\partial A}{\partial x}. \tag{24}$$

As a result, the explicit expressions for the electric and magnetic fields of the reflected pulse take the form

$$E_{\mathrm{ref}}(x,t)=(1+\beta_{ph})\gamma_{ph}\frac{q}{c}\Big\{A_{\mathrm{ref}}(x,t)+\mathcal{C}[\gamma_{ph}^2 v_{ph}(1+\beta_{ph}) \times(ct-x)]e^{i(1+\beta_{ph})\gamma_{ph}^2\omega_0(t-x/c)}\Big\}. \tag{25}$$

$$B_{\mathrm{ref}}(x,t)=iE_{\mathrm{ref}}(x,t). \tag{26}$$

The shape of the electric field in the reflected pulse is shown in Fig. 2. In the ultrarelativistic limit ($1+\beta_{ph}\approx 2$ and $1-\beta_{ph}\approx 1/2\gamma_{ph}^2$) the expressions of the vector potential and of the electric and magnetic fields can be reduced to the simpler form

$$A_{\mathrm{ref}}(x,t)=-q\,e^{-2q\gamma_{ph}(t-x/c)}\int_{-\infty}^{2\gamma_{ph}(t-x/c)}\mathcal{C}(\gamma_{ph}v_{ph}\tau)e^{(q+i\omega_0\gamma_{ph})\tau}d\tau, \tag{27}$$

and

$$E_{\mathrm{ref}}(x,t)=-iB_{\mathrm{ref}}(x,t) =2\gamma_{ph}\frac{q}{c}\Big[A_{\mathrm{ref}}(x,t)+\mathcal{C}(2\gamma_{ph}^2 v_{ph} \times(ct-x))e^{2i\gamma_{ph}^2\omega_0(t-x/c)}\Big]. \tag{28}$$

By evaluating the two terms in the square brackets in Eq. (28) numerically, it can be shown that they are of the same order. Thus, from Eq. (28) and from the definition of q below Eq. (18), we can conclude that the amplitudes of the electric and magnetic fields in the reflected pulse increase by a factor of order $\gamma_{ph}^{3/2}$, i.e., that its intensity is proportional to γ_{ph}^3. The frequency of the reflected pulse is up-shifted by the factor $2\gamma_{ph}^2$, as could be expected qualitatively from the considerations on the reflection of a plane wave from a relativistic ideal mirror in Sec. II. It can also be seen from Eq. (28) that the reflected pulse is compressed with respect to the initial one by the factor $2\gamma_{ph}^2$. In order to illustrate this effect, in Fig. 3 we present the electric fields of the initial and reflected pulses for $\gamma_{ph}=2$.

In a tenuous plasma the frequency up-shift of the reflected pulse, and its related compression, can be so large that they can provide a different mechanism of attosecond pulse generation. The Lorentz factor γ_{ph} of the wake field generated by a laser pulse in plasma is equal to $\gamma_{ph}\approx\omega_d/\omega_{pe}$, where ω_d is the frequency of the laser pulse that generates the wake plasma wave (see Ref. [4]). The frequency up-shift of the portion of the soliton field reflected by the wake plasma wave is $2\gamma_{ph}^2\omega_0\approx 2\gamma_{ph}^2\omega_{pe}\approx 2\gamma_{ph}\omega_d$. Thus, for a 1-μm wavelength laser pulse, corresponding to the critical plasma density $n_{\mathrm{cr}}\approx 10^{21}$ cm^{-3}, the factor $2\gamma_{ph}$ required to generate an attosecond reflected pulse must be of order 10^3, i.e., the density of the plasma must be sufficiently low, of the order of 4×10^{15} cm^{-3}.

If we take the dimensionless amplitude of the soliton (defined in terms of the soliton frequency in the laboratory frame) to be equal to a_0, the equivalent intensity of the e.m. field is given by $I_0=cE_0^2/4\pi$ and is equal to $I_0\approx(a_0/\gamma_{ph})^2\times10^{18}$ W/cm^2. After reflection at the electron density modulations in the wake plasma wave, the intensity of the reflected e.m. pulse becomes equal to $I_{\mathrm{ref}}\approx\gamma_{ph}^3 I_0$, i.e., $\approx a_0^2\gamma_{ph}\times10^{18}$ W/cm^2. Additional light intensification can occur because of the focusing effect that accompanies the reflection and that is due to the paraboloidal shape of a relativistically strong wake plasma wave, as demonstrated in Ref. [12], where the mirror interaction with a counterpropagating laser pulse in a tenuous plasma was studied. In the case of the interaction of the soliton with the wake field the enhanced scaling of the reflected wave intensity, $I_{\mathrm{ref}}\approx\gamma_{ph}^5 I_0$, leads to $\approx a_0^2\gamma_{ph}^3\times10^{18}$ W/cm^2. In Ref. [6] it was proven that within the one-dimensional (1D) approximation the dimensionless amplitude of a soliton, with a nonvanishing electron density inside the soliton, should be smaller than $3^{1/2}$. However, in the 2D and 3D cases the soliton amplitude can be substantially higher, as shown by the PIC simulation results (see Ref. [8]). For the plasma density and wake-field parameters discussed above corresponding to $\gamma_{ph}\approx10^3$, assuming the dimensionless soliton amplitude a_0 to be equal to 10, we obtain that the reflected pulse intensity approaches 10^{29} W/cm^2. This intensity is not far from the critical intensity (the so called Schwinger intensity) in quantum electrodynamics, $I_{\mathrm{Shw}}=5\times10^{29}$ W/cm^2. In principle, this allows us to consider the problem of the electron-positron pair production by the e.m. field: the Schwinger effect [13]. In addition, the e.m. field invariants, $\mathcal{F}=(\mathbf{E}_{\mathbf{ref}}^2-\mathbf{B}_{\mathbf{ref}}^2)/\mathbf{2}$ and $\mathcal{G}=(\mathbf{E}_{\mathbf{ref}}\cdot\mathbf{B}_{\mathbf{ref}})$ of the plasma wave pulse reflected and focused by the paraboloidal relativistic mirror are not equal to zero. We recall that, as shown in Ref. [25], electron-positron pairs can be produced in vacuum by a focused e.m. pulse.

IV. CONCLUSION

We have considered the interaction between a relativistic plasma soliton, generated by a circularly polarized e.m. wave, and a breaking-wake plasma wave. The electron density spike associated with such wave acts as a mirror flying with a relativistic velocity. We have computed the properties of the reflected pulse by first performing a Lorentz transformation to the reference frame where the wake plasma wave is at rest. In this frame we have used the reflection coefficient derived in Ref. [12] for the frequency components obtained by Fourier expanding the soliton vector potential. The form of the reflected pulse in this moving frame has been obtained by performing the inverse Fourier transform and the amplitude of the reflected pulse in the laboratory frame has then been obtained by successively performing the inverse Lorentz transformation.

The results of the one-dimensional investigation presented in this paper are partly based on idealizations of the plasma response and must be considered as a proof of principle of the possibility of exploiting a different mechanism for obtaining high-intensity attosecond pulses. For example, electrons may become trapped because the wake wave was assumed to be close to its breaking limit. Therefore, as a result of the interaction of the wake wave with the plasma inhomogeneity formed by the soliton, a few electrons can be injected into the wake wave as shown in Ref. [26].

Within these limitations we have shown that, in the case of the reflection of a plane wave by a mirror moving with velocity v_{ph}, the frequency is up-shifted by the factor $(1+\beta_{ph})/(1+\beta_{ph})\approx4\gamma_{ph}^2$ [$\gamma_{ph}=(1-\beta_{ph}^2)^{-1/2}$ is the Lorentz factor and $\beta_{ph}=v_{ph}/c$] in accordance with the Einstein formula [23]. We have shown that in a plasma this factor reduces to $2\gamma_{ph}^2$ in the case $\omega\approx\omega_{pe}$ (ω_{pe} is the Langmuir frequency). The reflected pulse is compressed with respect to the initial pulse by $(1+\beta_{ph}^2)/(1-\beta_{ph}^2)$, and the maximum amplitude of the electric field in the reflected pulse is increased by $\gamma_{ph}^{3/2}$ times. Thus a substantial increase in field intensity and frequency has been demonstrated.

In a tenuous plasma the frequency up-shift can be so high that it can provide a different mechanism of the attosecond pulse generation in the case when the wake field in the plasma is generated by a femtosecond laser. Then $\gamma_{ph}\approx\omega_d/\omega_{pe}$, where ω_d is the frequency of the driver laser pulse that generates the wake field. The frequency up-shift of the portion of the soliton reflected by the wake field is equal to $2\gamma_{ph}^2\omega_{pe}\approx2\gamma_{ph}\omega_d$. In order to generate an e.m. pulse in the attosecond range, the Lorentz factor must be equal to $\gamma_{ph}=5\times10^2$, which corresponds to a plasma density of the order of 4×10^{15} cm^{-3}.

If we take into account the additional intensification of the e.m. field that occurs in a three-dimensional configuration because of the paraboloidal form of the reflecting wake plasma wave when this wave is relativistically strong, the reflected pulse intensity can approach 10^{29} W/cm^2, i.e., it can approach the Schwinger limit for the electric field. This makes it possible to consider the production of electron-positron pairs by such fields (Schwinger effect).

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